Coordination and Free Riding: A Partnership Solution to the Common-Property Problem

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Abstract The common-property problem results in excessive mining, hunting, and extraction of oil and water. The same phenomenon is also responsible for excessive investment in R&D and excessive outlays in rent-seeking contests. We propose a "Partnership Solution" to eliminate or at least mitigate these excesses. Each of N players joins a partnership in the first stage and, in the common-property context, chooses his extraction effort in the second stage. Under the rules of a partnership, each member must pay his own cost of effort but receives an equal share (or a share partially reflecting relative effort) of the partnership's revenue. The incentive to free-ride created by such partnerships turns out to be beneficial since it naturally offsets the excessive extraction effort (or R&D investment or rent-seeking outlays) inherent in such problems. In our two-stage game, this institutional arrangement can, under specified circumstances, induce the social optimum in a subgame-perfect equilibrium: no one has a unilateral incentive (1) to switch to another partnership (or create a new partnership) in the first stage or (2) to deviate from socially optimal actions in the second stage. The game does have other equilibria, but the one associated with the "Partnership Solution" is strictly preferred by every player. Government can facilitate selection of this equilibrium by recommending the size of each partnership. Such "mediation" requires no enforcement power but can help coordinate expectations. Antitrust authorities should recognize that partnerships can have a less benign use. By organizing as competing partnerships, an industry can reduce output and raise prices, sometimes to the monopoly level. Even when the implicit threat of future punishment does not deter excessive output, the free-riding inherent in partnerships removes the temptation to overproduce.

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1 Introduction

In some fisheries in Japan, fishermen from several vessels share their catch. Their pooled output is sold through a common outlet and members of each partnership divide equally the resulting gross revenue, no matter how little someone has contributed. Such egalitarian catch-sharing among vessels is a prescription for free-riding; fortunately, there are multiple partnerships or the induced free-riding would be even greater.

Received economic theory cannot account for such partnerships. They appear to be a response neither to uncertainty nor to asymmetric information. Catch-sharing partnerships must have *some* advantage, however, since as of the census of 1988, 147 different fishing groups in Japan were engaging in such income pooling. To understand why such partnerships arise in the glass-shrimp industry, Platteau and Seki (2000) interviewed skippers and, when feasible, used more objective measures to validate their responses. They concluded that "The most prominent result emerging from this exercise is certainly the fact that stabilization of incomes was not mentioned a single time." Instead, the main motive appeared to be the reduction of congestion: "The desire to avoid the various costs of crowding while operating in attractive fishing spots appears as the main reason stated by Japanese fishermen for adopting pooling arrangements."

These Japanese fishermen appear to have rediscovered an ancient solution to the common-property problem. According to anthropologists, hunter-gatherer cultures that have survived to the modern era typically share their kill and work short hours; moreover, they consume enough quantity and variety to be characterized by one distinguished anthropologist as the "Original Affluent Society" (Sahlins, 1972). Kägi (2001) was the first to point out that these phenomena, which have been studied extensively but separately, may be connected: those hunter-gatherer cultures surviving to modern times owe their success to their practice of sharing the fish and game caught by groups of hunters since extensive sharing dulls hunting effort sufficiently to protect common property from over-exploitation.[1]

At the opposite end of the technological spectrum, individuals who form research joint ventures to share revenue from their discoveries may have hit upon the same solution. Without joint ventures, individuals vying for a patent awarded to the best innovation will invest too much (Baye and Hoppe, 2003) even taking account of the fact that the expected value of the winning patent grows with aggregate investment. Baye and Hoppe (Theorem 1) also demonstrate that such innovation tournaments are strategically equivalent to rent-seeking contests where the value of the prize increases with the total outlay. So, in the absence of prize-sharing within interest groups, rent-seeking outlays are also excessive (Chung, 1996).

Common property extraction provides a particularly relevant illustration of the same strategic considerations. In the absence of sharing, fishing effort is excessive (Gordon, 1954) even when account is taken of the fact that aggregate catch grows with aggregate effort. Sharing arrangements promote free-riding and, when effort would otherwise be excessive, this constitutes a social improvement.[2] Our analysis is motivated by the following question:

[1] "The literature on traditional hunter-gatherers provides ample evidence that work effort is extremely low in traditional societies and that natural resources are not overexploited but rather under-exploited" (Kägi, 2001, p.45). "We do not know whether traditional societies have introduced sharing consciously... Once introduced (or chosen by accident), however, it appears to be a stable means to regulate resource use." (Kägi, 2001, p.67).

[2] In an important contribution, Nitzan (1991) analyzed a contest for a fixed prize among rent-seeking individuals exogenously allocated to partnerships and found that aggregate rent-seeking outlays decline because of free-riding. Each partnership was assumed to use the same exogenous sharing rule of which our egalitarian rule is a special case. In other cases, his rule partially rewards an individual for making larger effort relative to other members of his group, which implicitly requires that group members costlessly monitor each other's efforts. We extend this original insight and show its implications for the common property and cartel problems. The case we examine is isomorphic to a rent-seeking contest where the prize is a strictly increasing function of aggregate outlays and where groups form endogenously (see footnote 8). Since the prize in Nitzan's contest is fixed, the partition generating the highest social welfare occurs when every player is in a single group. In our variable-prize case, putting every player in one group induces too little extraction effort and the social optimum instead occurs at an "interior" solution with the correct number groups. In Section 5, we show how our Partnership Solution easily generalizes when groups share according to Nitzan's rule provided the weight on relative effort is not excessive. Baik and Lee (2001) extend Nitzan's original model of group rent-seeking with a fixed prize by endogenizing group formation and the choice of the sharing rule. However, our analyses are quite different because there is no variable prize (the counterpart to our production) in their application. Consequently, it is efficient for everyone to join a single group, make no rent-seeking outlay, and share the fixed prize; in our context, the "prize" (production) would completely vanish in the absence of outlays (effort). Another key difference arises because they assume away team

under what circumstances can sharing agreements be used to restore the social optimum in (1) innovation tournaments, (2) rent-seeking contests with variable prizes, and (3) extraction from common properties?[3] For concreteness, we will focus on the last of these applications, the common-property problem.

Besides these beneficial uses, partnerships can also be used for a more sinister purpose. This possibility constitutes another reason for understanding when they can be successfully utilized. In the absence of partnership agreements, service providers in an industry can be expected to reap at most oligopoly profits. But if they can organize themselves into a collection of competing revenue-sharing partnerships (a common organizational form in some service industries), they can potentially reap monopoly profits. There is no need for interactions to be ongoing so that the prospect of a price war in the future deters the current temptation to expand output. The revenue-sharing inherent in a partnership structure *eliminates* the current temptation to expand output. Antitrust authorities should be aware that in industries where partnerships predominate, prices may approach monopoly levels even though competition among these partnerships is vigorous.

Suppose N individuals expend effort to extract a common resource and consider two extremes.[4] On the one hand, suppose everyone pays his own effort cost and acts independently; then, as is well known, aggregate effort will be excessive because of congestion externalities. On the other hand, suppose everyone must share the fruits of his labor equally with the other $N-1$ individuals while paying his own cost of effort; then aggregate effort will be insufficient because of free-riding. Each of these two extremes is a special case of the following arrangement: players partitioned into competing partnerships simultaneously choose effort levels, with each partnership's share of aggregate revenue equal to its share of

production, presumably because it has no plausible counterpart in their game.

[3]We clarify the strategic equivalence of these three applications in footnote 8 after introducing notation.

[4]In assuming that N is exogenous, we are abstracting from the entry of outsiders. In reality, partnerships have also been effective in protecting their own territories from outsiders. For an interesting analysis of how this has been accomplished see Acheson and Gardner's (2005) discussion of the Maine lobster fishery.

aggregate effort and every member of each partnership required to pay his own effort cost but to share equally with his colleagues the gross revenue he brings in. In the first of the extremes above, there are N "solo" partnerships while in the second, there is 1 "grand" partnership to which all N individuals belong.

Since too little effort occurs when there is 1 partnership and too much effort occurs when there are N partnerships, one might expect that aggregate effort increases with the number of partnerships. We verify this conjecture analytically and Schott et. al. (2005) confirm our prediction experimentally. Socially optimal effort can, therefore, be induced (or approximated if there are integer problems) if the N players partition themselves into an intermediate number of partnerships in such a way that each agent's tendency to work too hard is exactly offset by his tendency to free ride. We refer to this as the "Partnership Solution." While Pigouvian taxes or auctioned quotas could induce the same extraction effort as the Partnership Solution, resource users would receive larger aggregate surplus under the Partnership Solution as long as any of the revenue collected under these two alternative policies was diverted to general coffers rather than being redistributed to the N players.

In reality, of course, the Partnership Solution is viable if and only if each person in a given partnership has no incentive to switch to some other partnership (pre-existing or new). We refer to such partnerships as "stable." We investigate the stability of the Partnership Solution in a two-stage game where individuals simultaneously choose their partnerships at the first stage and choose their levels of effort at the second stage. We refer to each partition of players into partnerships as a "partnership structure." Whether a partnership structure is stable (formally, whether it is part of a subgame-perfect Nash equilibrium) turns out to depend on the advantages of team production over solo production. This follows since the principal source of first-stage instability is going into business for oneself (deviating unilaterally to a *new* partnership).

There are typically multiple subgame-perfect equilibria in this game. Within any equilibrium, every individual receives the same payoff. In the equilibrium associated with the Partnership Solution, however, the common payoff of every individual is largest. To facilitate selection of this "payoff-dominant" equilibrium, we use a powerful coordination device first proposed by Schelling (1960, p. 63 and p. 302). We assume that a "mediator" (the government) recommends publicly to all the players the strategy-profile underlying the Partnership Solution. By assumption, this mediator has no enforcement power of any kind. His only function is to focus the expectations of the players on the payoff-dominant equilibrium. In two-player experiments, Van Huyck et. al. (1992) have shown that subjects disregard such recommendations if they are not Nash or are Nash but not payoff-dominant; however, subjects follow the recommendations which are Nash and payoff-dominant 98% of the time. Brandts and MacLeod (1995) extend this experimental work to two-stage games and find that the mediator is equally influential in the selection of subgame-perfect equilibria.[5]

We proceed as follows. In the next section, we introduce our notation, define socially optimal effort, and show that this level of aggregate effort arises in the second-stage subgames if and only if a requisite number of partnerships forms at the first stage. In Section 3, we determine conditions sufficient for this Partnership Solution to be stable. In such circumstances, we assume that a mediator recommends the associated payoff-dominant subgame-perfect Nash profile, and players coordinate on this suggested profile. Section 4 generalizes the analysis while maintaining the assumption of egalitarian sharing within partnerships. Section 5 generalizes the Partnership Solution when the sharing rule ceases to be purely egalitarian but also takes account of relative effort. Section 6 concludes the paper.

[5] A useful discussion of these two papers is contained in Camerer (2003, 362-5). Further evidence that preplay communication can secure the payoff-dominant profile is provided by "cheap talk" experiments (Cooper et. al., 1992) where the players themselves communicate prior to play without benefit of a mediator. These cheap-talk experiments suggest that a cartel could coordinate on the payoff-dominant Partnership Solution even without a mediator.

2 Decentralization in a Two-Stage Partnership Game

To begin, we define the notation that will be used throughout this paper.

m_i = *number of members of group i*

x_{ik}= *effort level of agent k in group i*

Y_i^{-k} = *aggregate effort level of members of group i other than agent k*

X_{-i}= *aggregate effort of other groups*

X = *total effort level (sum of all agents' efforts)*

f(X) = *aggregate production function*

c = *constant marginal cost of effort*

n = *number of groups*

N = *total number of agents*

$A(\cdot) = \frac{f(X)}{X} =$ *average product*

$\bar{x}_i = \frac{\left(x_{ik}+Y_i^{-k}\right)}{m_i} =$ *mean effort level in group i*

β = *effort advantage of team production compared to solo production*

Until Section 4, we make the assumption standard in the common-property literature that the price of output is a constant (normalized to unity). In addition, we assume that (1) $A(X)$ is bounded, strictly positive, strictly decreasing, and twice continuously differentiable; (2) $A(0)-c>0$; and (3) that $A'(X)+XA''(X)<0$, holds for all $X \geq 0$. Following standard usage, we refer to this last inequality as the Novshek condition. Novshek (1985) showed this condition guarantees existence of a pure strategy Nash equilibrium in a standard Cournot game.[6] The condition is weaker than assuming that average product is strictly concave ($A''(X)<0$). We use it to establish that the first-order condition for socially optimal effort

[6] Gaudet and Salant (1991) showed that, in conjunction with other assumptions, it guarantees the *uniqueness* of that equilibrium.

has a unique solution and to sign several comparative-static results.

Socially optimal effort maximizes total net benefit ($X^* =$ argmax $X(A(X) - c)$). It must therefore satisfy the following first-order condition:

$$A(X^*) + X^* A'(X^*) - c = 0. \tag{1}$$

Since the Novshek condition holds, X^* is unique. This aggregate effort level is the goal we seek to achieve by decentralization through our Partnership Solution.

In the first stage of our two-stage game, agents choose a partnership to which to belong. Let $n \leq N$ denote the number of distinct groups specified by the agents and index these groups $i = 1, \ldots, n$. Then, in the second stage, agents simultaneously choose their effort after observing each agent's choice of group.[7] To verify that the partnership solution is subgame-perfect, we must show that it forms a Nash equilibrium in every subgame. We approach this through backwards induction, considering the problem of effort choice first.

2.1 Equilibrium Effort Choice in Second-Stage Subgames

Consider second-period subgames in which individuals grouped into partnerships simultaneously choose their effort levels.

An individual in group i would choose his own effort level (x_{ik}) taking as given the aggregate effort level of his colleagues in partnership i ($Y_i^{-k} = \sum_{l \neq k} x_{il}$) as well as the aggregate effort levels of the other partnerships (X_{-i}). Hence, he would maximize

$$\pi_{ik} = \underset{x_{ik}}{Max} \left\{ \frac{1}{m_i} \left[\frac{x_{ik} + Y_i^{-k}}{X} \right] \cdot f(X) - c x_{ik} \right\},$$

[7]The assumption that agents observe the composition of their partnership before exerting effort seems plausible; however, it is not innocuous. If effort choices had to be made without observing the partnership partition, then there would be no pure-strategy Nash Equilibria.

where $X = x_{ik} + Y_i^{-k} + X_{-i}$ and m_i is the number of partners in his group.[8] This is equivalent to maximizing:

$$m_i \pi_{ik} = \left(x_{ik} + Y_i^{-k} \right) \cdot A \left(x_{ik} + Y_i^{-k} + X_{-i} \right) - m_i c x_{ik}. \tag{2}$$

To find the best response of member k in partnership i, we differentiate the objective function (2) with respect to x_{ik} and substitute $X = x_{ik} + Y_i^{-k} + X_{-i}$ to arrive at the following N first-order conditions:

$$\frac{1}{m_i} \left[A(X) + \left(x_{ik} + Y_i^{-k} \right) \cdot A'(X) \right] = c \text{ for } i = 1, \ldots, n \text{ and } k = 1, \ldots, m_i. \tag{3}$$

Each first-order condition in (3) clarifies why player i reduces his effort in a multiperson partnership compared to his effort operating solo, for unchanged effort of the other $N-1$ players. There are two effects, each of which leads him to reduce his effort: the "internaliza-

[8] We can now formally show the connection between the common-property problem and the two other applications discussed in the introduction. Let x_{ik} denote the rent-seeking outlay of player k in group i, Y_i^{-k} denote the outlays of his partners, X denote the total outlay of all contestants, and $f(X)$ denote the prize. Then, assuming the probability that a group wins the prize is proportional to its rent-seeking outlays and the prize is divided evenly among members of the winning group, the foregoing objective function of player i is simply his expected payoff from an outlay of x_{ik}. Similarly, consider an innovation tournament in which player k in group i makes research effort x_{ik}. Following Baye and Hoppe (2003), conceive of each researcher's discovery effort as his drawing ideas worth \$y if awarded a patent ($y \in [0, U]$, where U is an exogenous highest conceivable value) from a differentiable probability distribution $G(y)$. Let x_{ik} denote the integer number of times ideas are drawn with replacement from the distribution. If each draw costs the researcher c, then researcher k's cost would be cx_{ik}. Moreover, if the prize goes to the partnership with the highest realized draw, then the probability that partnership i wins equals its aggregate number of draws as a percentage of the total number of draws of all players in the tournament (the factor in square brackets). It remains to show that—whoever wins—the expected value of the prize is a strictly increasing, strictly concave function of the total number of draws. This is both intuitive and straightforward to show (see Theorem 1 of Baye and Hoppe, 2003): $\int_{y=0}^{U} yXG^{X-1}(y)G'(y)dy = 1 - \int_0^U G^X(y)dy = f(X)$, where the first integral is the expected value of the prize when there are X draws from distribution $G(\cdot)$ and the largest of them is designated y. It is straightforward to verify by differentiating that the resulting expected value of the winning prize ($f(X)$) is a strictly increasing and concave function of the aggregate research effort as our analysis assumes.

tion effect" and the "diversion-of-benefits effect." First, since in a multiperson partnership, player i receives a *share* of the receipts generated by his partners, he would refrain from imposing as large a negative externality on them as he would if he operated solo. That is, the first factor in the second term is larger by Y_i^{-k} than it would be if he operated solo. This "internalization effect" would induce him to reduce his effort in a multiperson partnership even if $c = 0$ but the effect would disappear if under the rules of the partnership he received nothing from his partners. Second, since in a multiperson partnership, player i must *relinquish* a share of the benefits of his effort but must pay the full cost of generating them, he would reduce his effort. That is, the second factor in the last term is $m_i > 1$ times as large as it would be if he were operating solo. This "diversion-of-benefits effect" would persist even if his partners were not, like him, required to share their own benefits but would disappear if $c = 0$.

Consider any solution to the N equations in (3) and the 2 equations defining X and Y_i^{-k}. Notice that if the efforts within any partnership are rearranged without altering their sum that each of these $N + 2$ equations still holds. Formally, therefore, there are multiple Nash equilibria in the final stage of our game, with the payoff to player k lower in those equilibria in which he undertakes a larger effort. In exactly these circumstances, Benoit and Krishna (1985) have shown that many first-stage configurations can be supported as subgame perfect equilibria. While exploiting this aspect of our problem would allow us to show that the Partnership Solution is stable in a wider set of circumstances, we refrain from doing so. We regard the multiplicity of equilibria in the second stage of our game as an artifact of our assumption that each of our homogeneous agents has a linear cost function; introducing even the slightest convexity would eliminate all but the symmetric Nash equilibrium. To avoid discussing partnership solutions that would disappear if even the slightest convexity was introduced, we therefore assume as a "refinement" that agents anticipate at the first stage that efforts will be shared equally in any partnership they join.[9]

2.2 Partnership Effects on Effort Choice

Since the first-order conditions in (3) depend only on the aggregate effort of partnership i and not on the efforts of its individual members, we can re-write the conditions in terms of the mean effort in partnership i ($\bar{x}_i$). Rewriting the first factor of the second term gives us:

$$A(X) + m_i \bar{x}_i \cdot A'(X) - cm_i = 0, \quad \text{for } i = 1, \ldots, n. \tag{4}$$

These n equations plus the equation $X = \sum_{i=1}^{n} m_i \bar{x}_i$ uniquely determine the n mean effort levels $\{\bar{x}_i\}_{i=1}^{n}$ and X. We can solve (4) for $\bar{x}_i$, the mean effort level in group i:

$$\bar{x}_i = \left(\frac{1}{-A'(X)}\right)\left(\frac{A(X)}{m_i} - c\right). \tag{5}$$

If partnerships of different sizes form at the first stage, then their mean effort levels will differ at the second stage. In particular,

Proposition 1 *In any equilibrium, strictly larger groups have strictly smaller mean effort levels.*

Proof: As (5) reflects, the strictly positive mean effort level at the i^{th} partnership can be represented as the product of two positive factors. The second factor will be smaller at a partnership with a larger number of members (m_i) while the first factor will be the same for all the partnerships. Hence, the larger the partnership the smaller the mean effort.■

Intuitively, the larger the group, the more free-riding occurs within it.

[9]To clarify what we are ignoring, suppose anyone deviating to a different partnership at the first stage anticipated that, as "hazing" reserved for new members, his new colleagues would undertake no effort and would make him carry the entire effort burden of the partnership by himself. The anticipated situation would deter his deviation and is technically "credible" since it is a Nash equilibrium. However, since the underlying multiplicity of equilibria would disappear the moment any convexity is added to the cost function of every agent, we ignore Partnership Solutions supported in this way.

Next we verify that *aggregate* effort in the second-stage depends only on the *number* (n) of groups formed at the first stage and not on the distribution of agents among the different groups:

Proposition 2 *Aggregate effort (X) in the second stage depends only on the number of groups formed in the first stage and not on the size of those groups.*

Proof: Adding together the n first-order conditions in (4), we obtain the following condition:[10]

$$nA(X) + XA'(X) - cN = 0. \tag{6}$$

Thus aggregate effort (X) induced in the Nash equilibria of second-stage subgames depends only on the number of groups formed at the first stage and not on the specific partition. ■

A monotonic relationship exists between the number of partnerships formed at the first stage and the aggregate effort expended at the second stage.

Proposition 3 *If the number of groups formed at the first stage is strictly larger, the aggregate effort level at the second stage is strictly larger.*

Proof: Differentiating (6) implicitly, we obtain:

$$\frac{dX}{dn} = \frac{A(X)}{-[(n+1)A'(X) + XA''(X)]} > 0,$$

where the inequality follows from $A(X) > 0, A'(X) < 0$, and the Novshek condition.■
Since aggregate effort in our game is a continuous, strictly increasing function of the number of groups formed at the first stage and since $n = 1$ induces too little aggregate effort and $n = N$ generates too much, some unique intermediate number of groups will (if we

[10]Our proposition reinterprets the result in Bergstrom and Varian (1985) that, in an interior equilibrium of a Cournot oligopoly model with constant marginal costs, aggregate output depends only on the sum of the marginal costs.

provisionally ignore integer constraints) induce the socially optimal level of effort at the second stage. We can find this number by plugging X^* into (6) and then solving for n^*.

Proposition 4 *If* $n^* = \frac{c(N-1)}{A(X^*)} + 1$ *groups form at the first stage, then the aggregate effort chosen in the Nash equilibrium of the second stage will be socially optimal.*

Proof: Substitute $n^* = \frac{c(N-1)}{A(X^*)} + 1$ and X^* into (6). This gives us:

$$\left(\frac{c(N-1)}{A(X^*)} + 1 \right) A(X^*) + X^* A'(X^*) - cN = 0.$$

Simplifying, we obtain:

$$A(X^*) + X^* A'(X^*) - c = 0$$

which is the same as (1), the condition defining X^*.■

Proposition 4 implies that whenever $c = 0$, the social optimum is achieved by putting everyone into a single partnership ($n = 1$), while for $c > 0$, the optimum is achieved by dividing them among several partnerships. To understand these results, suppose each of the N players joins a single partnership and everyone's effort equals $1/N^{th}$ of the aggregate profit-maximizing level. If costs are zero, aggregate revenue would also be maximized so that if someone marginally reduced his effort unilaterally, total revenue would not change nor would his share of it. Since deviating is unprofitable, the social optimum is achieved as a Nash equilibrium. On the other hand, if everyone is assembled into a single partnership and asked to undertake $1/N^{th}$ of the socially optimal effort when $c > 0$, there is an incentive to reduce effort. In this case, such a reduction does not affect aggregate profit because aggregate gross revenue and aggregate costs fall by equal amounts. However, since all of the cost savings accrue to the deviator while only $1/N^{th}$ of the aggregate revenue loss is borne by him, he has a strict incentive to reduce his effort and it is no longer possible to achieve the social optimum with a single partnership.

When $c > 0$, therefore, the Partnership Solution requires dividing the N players among *several* partnerships. Suppose again, each player undertakes $1/N^{th}$ of the socially optimal effort. If someone marginally reduced his effort unilaterally, he would save the same costs as before but now would incur a larger loss in revenue both because his partnership's losses are larger and because he bears a larger share of those losses.

In the Partnership Solution ($n = n^*$), the social optimum is achieved as a Nash equilibrium because each person fully anticipates the *social* consequences of his deviation. If anyone increased his effort, he would incur the entire cost increase. Moreover, since the gross revenue of his colleagues would expand by exactly as much as the gross revenue of noncolleagues would contract, the deviator's share of his partnership's increased revenue would just equal the aggregate gain in revenue from his deviation.[11]

To illustrate, suppose that $N = 12$ players earn their livelihood working in an activity plagued by a congestion externality. Assume aggregate production (and hence aggregate revenue) is $f(X) = 19X - X^2$, where X represents aggregate effort. Suppose that the cost per unit of effort is $c = 3$. Socially optimal effort is $X^* = 8$ since overall profits are concave and at that point a contraction of effort by one unit would reduce both cost and revenue by \$3. Since $N = 12$, Proposition 4 implies that:

$$n^* = \frac{3(11)}{19 - 8} + 1 = 4.$$

That is, if the 12 players divide into 4 partnerships, each with 3 members, then in equilibrium each effort level is 8/12 and the resulting aggregate effort is socially optimal. Anyone

[11]More precisely, since at the social optimum the planner equates to marginal cost the marginal revenue from additional effort while in the Partnership Solution each player equates to that same marginal cost the increase in gross revenue he would receive through the partnership agreement if he marginally increased his effort, the following equality must hold:$[A + XA'] = \frac{1}{m_i}[A + (X - X_{-i})A']$. But this is equivalent to the following: $\frac{m_i-1}{m_i}[A + (X - X_{-i})A'] + X_{-i}A' = 0$. The first term can be interpreted as the aggregate increase in gross revenue of the deviator's colleagues and the second term can be interpreted as the exactly offsetting aggregate decrease in the gross revenue of players outside the partnership.

increasing his effort one unit would incur a cost increase of c =\$3 but a revenue increase of the same amount ($\frac{1}{3}[A(8) + 3\frac{8}{12}A'(8)]$ =\$3). Since each colleague of the deviator also gains \$3, their collective gain is +\$6, exactly offsetting the collective loss of the 9 agents outside the partnership ($9\left[\frac{8}{12}A'(8)\right]$ = -\$6). If n^* is not an integer, the Partnership Solution can only approximate the maximum social surplus.[12] Henceforth, we assume that n^* is an integer.

3 Equilibrium Partnership Choice in the First Stage

To implement the Partnership Solution, we assume that a mediator recommends to the players that:

1. They partition themselves into n^* groups in such a way that no two groups differ in size by more than one member.[13]

2. Each player, after observing the number of groups and inferring anticipated aggregate effort from (6), sets his own effort level equal to the mean effort of his group as given in (5).

No player would have an incentive to deviate unilaterally from the mediator's recommendation in Step 2 since he would anticipate that the others are making the recommended efforts then and these recommendations form a Nash equilibrium. But does any agent have an incentive to deviate in the first stage from the partnership recommended by the mediator

[12] Suppose in the previous example that $N = 8$ instead so that, while $X^* = 8$ as before, now $n^* = \frac{32}{11} = 2.91$. There are two possible integer solutions, $n = 2$ or $n = 3$. If $n = 3$, then we find that $X = 8.25$ which yields a social product of 63.94, while if $n = 2$ then $X = 4.67$ and the social product is 52.89. So $n = 3$ is optimal given the integer constraint. Setting $n = 3$ allows society to obtain 99.9% of the maximal social product, while only 40% can be achieved under the common property solution (where $n = N$).

[13] This can always be done. Imagine the n partnerships arranged around a table and each of the N agents dealt sequentially to the partnerships like cards in a deck until all N agents have been dealt out. At most, some partnerships would have one more agent than other partnerships.

given that he anticipates sharing the workload of that partnership equally at the second stage? If not, we will have established one way to implement the Partnership Solution. If so, we will have established that the Partnership Solution cannot be implemented.

Deviations at the first stage fall into two categories: (1) an agent can abandon the colleagues in his prescribed group for the members of some other group or (2) he can abandon his prescribed group to go into business for himself. As the following proposition shows, the first type of deviation is never advantageous.

Proposition 5 *If groups differ in size by at most one member, then no one can strictly improve his payoff by joining another group.*

Proof: First note that, from Proposition 2, a deviation which maintains the number of groups formed at the first stage will not alter aggregate effort (X^*) exerted at the second stage. Second, each player in group i anticipates a payoff of (π_i) is:[14]

$$\pi_i = \bar{x}_i(A(X^*) - c). \tag{7}$$

This is strictly increasing in $\bar{x}_i$ since $A(X^*) - c > 0$. Each member's payoff is larger in groups with a larger mean level of effort. Proposition 1 tells us that a group with a smaller number of members will have a larger mean effort since its smaller size will discourage free-riding. Hence, the only way to strictly increase one's anticipated payoff by defecting to another group is to switch to a group which, even *after* the defector is added, is strictly smaller than his original group. But there are no such opportunities to increase one's anticipated payoff if groups initially differ in size by at most one member.■

Consider the second type of deviation: an agent deviates to form a new, singleton, group. Whether this is profitable or not depends upon the disadvantage of solo production compared to team production. The literature on the theory of the firm identifies the disadvantages of organizing multi-agent firms. Such firms are rife with incentive problems to which single-agent firms are immune. But, since multi-agent firms abound, there must be a

[14]To see this, begin with the objective function (2) and see that, if player k makes effort x_{ik} in a group with mean effort $\bar{x}_i$ when aggregate effort is X, then his payoff is: $\pi_{ik} = \bar{x}_i A(X) - c x_{ik}$.

countervailing advantage to such arrangements—individuals working in teams must be able to produce more output per man-hour than those working alone.[15] Following the literature on team production, therefore, we consider the possibility that a team can produce more than an individual working by himself the same number of man-hours; in extreme cases, a team may be necessary in order to produce at all.

Suppose that to duplicate the efforts of 1 man-hour of team effort, a single individual must work $1/\beta$ hours, for $\beta \in [0, 1]$. Then, if we continue to express effort in man-hours of team effort, the marginal cost of effort for an individual working alone would be $\frac{1}{\beta}c$.

Partition the N players into n groups in such a way that no two groups differ in size by more than 1 member. For any n there is a unique partition that satisfies this restriction.[16] In the case where some partnerships are one member larger than others, these larger partnerships will generate more free-riding in the equilibrium of the second stage (Proposition 1). Anticipating lower payoffs in the second stage, every member of a larger partnership would have a stronger incentive to deviate to a solo partnership at the first stage. Let $g(n, \beta)$ denote the gain a member of a larger partnership would achieve by setting up his own partnership. If $g(n, \beta) \leq 0$ then he has no incentive to deviate and *a fortiori* neither does any member of a smaller partnership; hence the partition under consideration is stable. If, however, $g(n, \beta) > 0$ then he has an incentive to deviate and the partition under consideration is unstable. By analyzing properties of the $g(\cdot, \cdot)$ function, we show below that for any n, including n^*, there is a unique $\beta(n) \in (0, 1]$ such that the Partnership Solution is stable for all $\beta \leq \beta(n)$.

[15] Alchian and Demsetz (1972) were the first to emphasize the importance of team production in the theory of the multi-person firm and their insights have now percolated down to undergraduate treatments of that theory. For an extensive discussion, consult the textbooks by Eaton et. al (Chapter 19, 2002) and Campbell (Chapter 2.5, 1995).

[16] Recall the analogy to dealing cards in footnote 10.

3.1 Team Production is Essential ($\beta = 0$)

In many applications, "it takes two workers to perform a given task" (Holmstrom and Tirole, p. 67). That is, solo production is infeasible. For example, no matter how hard a person works he/she cannot catch a whale by himself; nor can he/she stay awake every day and night of his medical career to help patients with their medical emergencies. In other applications deviating to solo groups may be illegal since many partnership agreements contain 'non-compete' clauses which prevent an individual, when leaving a partnership, from competing in the same market as the group he is leaving.[17]

Whenever solo production is infeasible, $g(n, 0) < 0$ and we can conclude:

Proposition 6 *When solo production is infeasible, the Partnership Solution solves the common-property problem.*

Proof: As we have verified, no unilateral deviation to an existing partnership is strictly advantageous to any agent. Moreover, since $g(n, 0) < 0$, no deviation to a solo partnership is profitable for any n, including n^*. ■

3.2 Solo Production Is Feasible ($\beta \in (0, 1]$)

If solo partnerships are legal and feasible, we must investigate further. If $\beta < 1$, then social welfare can never be maximized as long as any solo partnership is involved. For, if there are any solo partnerships, then even if in equilibrium optimal effort (X^*) results, the cost of achieving it will strictly exceed cX^*, which a planner could achieve just by assembling a team of all N players and commanding that level of effort. So we assume that $n = 1, 2, \ldots, \lfloor N/2 \rfloor$ partnerships, where $\lfloor Z \rfloor$ denotes the greatest integer less than or equal

[17] Various courts have upheld such clauses, including the Georgia Supreme Court in Rash v. Toccoa Clinic Med. Assoc., 253 Ga. 322, 320 S.E.2d 170 (1984).

to Z. For example, if $N = 15$, there are at most $\lfloor 15/2 \rfloor = 7$ partnerships: six with two members and one with three members.

Equation (6) implicitly defines the aggregate effort which would result from n partnerships, each of which has two or more members. Denote the aggregate effort implicitly defined by this equation as $X(n)$. If $X(\lfloor N/2 \rfloor) \geq X^*$, then the Partnership Solution can potentially achieve the first best by generating more free riding and thereby bringing effort down toward X^*.

Denote the payoff of a potential deviator, prior to his deviation, as π^C and his payoff after going solo as π^D. π^C is independent of β. A partner who deviates, therefore, gains $g(n, \beta) = \pi^D - \pi^C$. His gain from going solo, his effort, everyone else's effort, and aggregate effort, will depend on the parameter β. Define $\underline{\beta}$ such that for any $\beta > \underline{\beta}$, the deviator going solo would make strictly positive effort while for any smaller β he would make zero effort. When $\beta \in [0, \underline{\beta}]$, the deviator would receive a zero payoff ($\pi^D = 0$) following his deviation. Hence, $g(n, \beta) = g(n, 0) = -\pi^C < 0$ for any $\beta \in [0, \underline{\beta}]$. When $\beta \in (\underline{\beta}, 1]$ the consequences of one agent's going solo are described by the four variables π^D, X, X_{-1}, and $\bar{x}_1$ which are defined by equations (8)-(11) below, where for simplicity we assign the index "1" to the deviator's solo partnership (and therefore denote his effort as $\bar{x}_1$ and the aggregate effort of all others as X_{-1}):

$$\pi^D = \bar{x}_1 (A(X) - \frac{c}{\beta}) \tag{8}$$

$$A(X) + \bar{x}_1 A'(X) - \frac{c}{\beta} = 0 \tag{9}$$

$$nA(X) + X_{-1} A'(X) - (N-1)c = 0 \tag{10}$$

$$\bar{x}_1 + X_{-1} = X. \tag{11}$$

Equation (10) is obtained by adding up the first-order conditions of the n original partnerships after effort levels have adjusted in response to the deviation.

Proposition 7 *$g(n, \beta)$ is a continuous function of β for any β in $(\underline{\beta}, 1]$.*

Proof: Since π^C is independent of β, it is sufficient to show that π^D is continuous in β. Use (11) to eliminate X from (8)-(10). Equation (10) does not involve β. Given the Novshek condition, $(n+1)A' + X_{-1}A'' \neq 0$; therefore, the implicit function theorem insures that, in a neighborhood of any solution $(\bar{x}_1, X_{-1}, X)$ induced by $\beta \in (\underline{\beta}, 1]$ we can write (10) as $X_{-1} = f(\bar{x}_1)$ where $f(\cdot)$ is a continuous function with derivative $f' = -\frac{nA'+X_{-1}A''}{(n+1)A'+X_{-1}A''} \in (-1, 0)$. Equation (9) does involve β. Replace X_{-1} in this equation by $f(\bar{x}_1)$. Given the Novshek condition and $A' < 0$, $(1+f')(A' + \bar{x}_1 A'') + A' \neq 0$; therefore, the implicit function theorem insures that we can write (9) locally as $\bar{x}_1 = h(\beta)$ for some continuous function $h(\cdot)$ with derivative $h' = -\frac{c/\beta^2}{A'+(1+f')(A'+x_1A'')} > 0$. Substituting both of these continuous functions into (8), we obtain:

$$\pi^D(\beta) = h(\beta) \left[A \left(h(\beta) + f(h(\beta)) \right) - c/\beta \right].$$

Since $A(\cdot)$ is continuous and since sums, products, and compositions of continuous functions are continuous, π^D is a continuous function of β in a neighborhood of any solution $(\bar{x}_1, X_{-1}, X)$ induced by $\beta \in (\underline{\beta}, 1]$. Given this conclusion, there can be no $\beta \in (\underline{\beta}, 1]$ where π^D is discontinuous. It follows that $g(n, \beta)$ is continuous for β in the open interval $(\underline{\beta}, 1]$. ■

Since $g(n, \beta) = \pi^D(\beta) - \pi^C$, we can differentiate to obtain the partial derivative, $g_\beta(n, \beta)$ anywhere in the open interval:

Proposition 8 *$g_\beta(n, \beta) > 0$ for any β in $(\underline{\beta}, 1]$.*

Proof: Since $\bar{x}_1 > 0$ for any β in $(\underline{\beta}, 1]$, $h(\beta) > 0$. Recall that $A' < 0$. Differentiating our expression for $g(n, \beta)$ and using (9) to simplify (an application of the envelope theorem) we conclude that:

$$g_\beta(n, \beta) = h'[A + hA' - c/\beta] + h[A'f'h' + c/\beta^2] = h[A'f'h' + c/\beta^2] > 0$$

for any β in $(\underline{\beta}, 1]$.■

We have shown that $g(n, \beta)$ is continuous and strictly increasing in β in the interval $(\underline{\beta}, 1]$ and $g(n, \beta) = -\pi^C$ for $\beta \in [0, \underline{\beta}]$. The following lemma establishes that there is no discontinuity at the boundary $\beta = \underline{\beta}$.

Lemma 1 *The function $g(n, \beta)$ is continuous in β at the point $\underline{\beta}$.*

Proof: Since $g(n, \beta) = -\pi^C$ for $\beta \in [0, \underline{\beta}]$, it suffices to verify that $\lim_{\beta \downarrow \underline{\beta}} g(n, \beta) = \lim_{\beta \downarrow \underline{\beta}} (\pi^D - \pi^C) = -\pi^C$. But this follows from (8) since $\lim_{\beta \downarrow \underline{\beta}} \bar{x}_1 = 0$ and A is bounded. ■

We can therefore, conclude:

Proposition 9 *If the partition indexed by n is stable for some β, then it is stable for all smaller β.*

Proof: This follows from Proposition 8. ■

We now use the results above to prove the existence and uniqueness of a 'threshold' $\beta(n)$ which separates stable from unstable partitions.

Proposition 10 *For any $n \leq \lfloor N/2 \rfloor$, there exists a unique $\beta(n) \in (\underline{\beta}, 1]$ such that for any $\beta < \beta(n)$, the partition indexed by n can be supported as a subgame-perfect equilibrium, while for $\beta > \beta(n)$ the partition can never be supported.*

Proof: For any given n, suppose that at $\beta = 1$, $g \leq 0$. Then that partition can be supported as an subgame-perfect equilibrium for any $\beta \in (0, 1]$ and we can define $\beta(n) = 1$. Now suppose that at $\beta = 1$, $g > 0$. Then by continuity (Proposition 7), there will exist one or more roots, $\beta \in (0, 1)$, such that $g(n, \beta) = 0$. Denote any root as $\beta(n)$. Uniqueness of $\beta(n)$ then follows since g is strictly increasing (Proposition 8).■

This makes precise the intuitive notion that the socially optimal partnership partition is stable if team production is "sufficiently advantageous": solo production need not be infeasible ($\beta = 0$) but β can not be strictly larger than $\beta(n^*)$. Alternatively, for any given β, $\beta(n)$ also defines partnership partitions which are stable:$\{n : \beta(n) \leq \beta\}$.

Is the Partnership Solution *always* stable even when team production confers no advantage whatsoever ($\beta = 1$)? A single counterexample suffices to eliminate this possibility. Recall the example introduced at the outset where $N = 12$ producers in an industry, each with constant marginal cost of $c = 3$, face an inverse demand curve of $P = 19 - X$ and attempt to achieve monopoly profits by dividing into $n = 4$ partnerships of equal size. It is easily verified that for any $\beta \leq .39$, full monopoly profits (\$64) can be achieved, but for $\beta > .39$ the configuration of four partnerships is unstable. There remains the possibility that for at least some example satisfying our assumptions, the partnership solution is stable even in the absence of advantages to team production. This seems unlikely since, in general, a partition with fewer than n^* partnerships can never be stable.[18]

4 Generalizations Using the Same Sharing Rule

Until now, we have assumed that no costs were *shared* within a partnership. We have also assumed that no individual or partnership had the power to change the price of output. Even in the case of the Japanese fishermen, however, neither of these simplifications is entirely realistic. According to Platteau and Seki, some costs are shared among the members of each partnership. Moreover, although the primary reason why Japanese fisherman partition themselves into partnerships is to reduce congestion, a secondary reason is to raises prices: "Fishermen believe that by limiting effort they can cause fish prices to rise." Platteau and Seki's statistical analysis of price data confirmed this effect.

To relax these two maintained assumptions, we need only reinterpret our previous analysis. In addition, we show how partnerships can be valuable as a way to increase payoffs even when the first-best is unattainable.

[18]When $\beta = 1$, $n < n^*$ is never stable. Recall that the only partitions we need consider are those where partnerships differ by at most one member and it is anticipated that effort is shared equally among the partners. Pick a partnership and designate someone as a potential deviator. Before going solo, he would anticipate earning exactly the same payoff as everyone else in his partnership; after going solo, he would earn at least as much as his ex-partner(s) since he would eliminate free-riding and $\beta = 1$. If, for the sake of argument, he did not strictly benefit from going solo then (1) the payoff of his ex-partners would likewise not increase and (2) the payoff of everyone else must strictly decrease. But then the sum of the payoffs would strictly decrease which contradicts the fact that the aggregate profit function is increasing in the number of partnerships to the left of n^*. An analogous argument establishes the strict profitability (when $\beta = 1$) of a "marginal" deviation in the neighborhood of n^* partnerships. See our earlier working paper: Heintzelman, Salant, and Schott (2004).

Suppose we partition N homogeneous agents into n payoff-sharing groups indexed by i, each playing a simultaneous-move game. Assume agent k in group i chooses x_{ik} to maximize $\frac{1}{m_i}\left[x_{ik}+Y_i^{-k}\right]\cdot G(x_{ik}+Y_i^{-k}+X_{-i})-cx_{ik}$. If we make the same assumptions about $G(X)$ that we made about $A(X)$ then we will get the corresponding results. So assume that (1) $G(X)$ is strictly positive, strictly decreasing, and twice continuously differentiable; (2) $G(0)-c>0$; and (3) the Novshek (1985) condition, $G'(X)+XG''(X)<0$, holds for all $X\geq 0$. These assumptions are sufficient to insure the existence of a pure-strategy Nash equilibrium in the simultaneous-move game. Because $G(\cdot)$ is downward-sloping, there is a negative externality: agent k is adversely affected by increases in X_{-i}. We have derived conditions sufficient for the aggregate payoff, $X(G(X)-c)$, to be maximized: provided $n^*\leq\lfloor N/2\rfloor$ and $\beta<\beta(n^*)$, the optimum can be achieved by setting up n^* partnerships differing in size by at most one member.

Suppose $G(X)=A(X)-K$, where K denotes cost per unit effort for those costs shared within the partnership. Then the Partnership Solution maximizes producer surplus. Since price is constant, this maximizes social welfare as well.

Next suppose $G(X)=P(f(X))A(X)-K$, where $P(\cdot)$ is the industry price when aggregate output $f(X)$ is put on the market. This generalization fits the case of the Japanese fishermen, who share some but not all costs and who use their partnerships not merely to curb congestion but to raise price. Again, the Partnership Solution maximizes producer surplus.

Finally, suppose $G(X)=P(X)-K$, where X is now interpreted as *output* and K (respectively, c) as the cost per unit *output* rather than effort, which is shared (respectively, not shared) within the partnership. In this case, there is no congestion externality and hence no common property problem. The Partnership Solution can be used to curb excessive output and permits a cartel to maximize profits without any need for supergame strategies. An outside observer would simply see an industry with a collection of firms organized as partnerships in competition with one another.

In cases where $\beta > \beta(n^*)$, the advantages of team production are insufficient to achieve the first-best using the Partnership Solution. In such cases, a generalization of the Partnership Solution can nonetheless lead to a *second-best* equilibrium with a large increase in the aggregate payoff. To illustrate, recall the example where $N = 12, c = 3$, and $G(X) = 19-X$. In that case $n^* = 4$ and $\underline{\beta} = .39$. Suppose as in our earlier example that $\beta = .56 > .39$. Then dividing the agents into four partnerships of equal size is not feasible since each member would have an incentive to go solo. However, if the 12 agents are divided into six partnerships of equal size, then industry profit is \$54.12—not the first-best level of \$64 but approximately *triple* the result in the oligopoly (or common property) solution.

5 Generalizing the Sharing Rule

In the rent-seeking game of Nitzan (1991), the group winning the fixed prize divides it using a sharing function S_{ik} which is an exogenous weighted average of our egalitarian rule and a rule rewarding relative effort:

$$S_{ik} = (1-a)\frac{1}{m_i} + a\frac{x_{ik}}{(x_{ik} + Y_i^{-k})}, \tag{12}$$

where $a \in [0,1]$.[19] We have so far restricted attention to the egalitarian case ($a = 0$) because it is simpler and does not require that members perfectly and costlessly monitor each other's efforts. When such monitoring is possible, however, we can show that our results generalize. For any $a \in (0,1]$, it is straightforward to verify that any partition of players into groups generates a Nash equilibrium in the second stage in which every member of a larger group makes smaller effort and receives a smaller payoff. Therefore, as in our model, the groups

[19] Sen (1966) previously used this weighted-average rule when discussing sharing within cooperatives.

formed at the first stage will differ in size by at most one member.[20]

If member k of group i makes effort x_{ik}, his payoff is $S_{ik}(x_{ik}+Y_i^{-k})A(X)-cx_{ik}$. Summing each player's best reply,[21] we obtain an equation linking effort to the number of groups and the weight on the relative effort component of the sharing rule:

$$kA(X) + XA'(X) - Nc = 0, \tag{13}$$

where $k = aN+(1-a)n$. If sharing is purely egalitarian ($a = 0$), this reduces to equation (6). As before, aggregate effort is strictly increasing in the number of groups and independent of the distribution of players among these groups. However, with this more general sharing rule, increasing the weight (a) on relative effort provides a second channel through which to stimulate free riding and reduce aggregate effort.

Using our egalitarian sharing rule ($a = 0$), we previously found that aggregate effort could be reduced to the socially optimal level (X^*) by reducing the number of groups from N to n^* defined in Proposition 4. Equation (13) can be used to generalize the Partnership Solution when the sharing rule puts more weight on relative effort. Since n^* groups would generate socially optimal effort (X^*) when $a = 0$, the number of groups ($n(a)$) which would achieve the social optimum when more weight is put on relative effort is:

$$n(a) = \frac{n^* - aN}{1-a}. \tag{14}$$

As a increases, $n(a)$ decreases until, at $a^* = \frac{n^*-1}{N-1}$, a single group is required to achieve X^*. Partnerships can solve the common property problem only if $a \in [0, a^*]$.[22] Denote this

[20] As with the egalitarian rule, there will also be an incentive to deviate by going solo unless solo production is either impossible or sufficiently costly compared to production with teams of two or more players.

[21] The best reply (x_{ik}) solves the following equation: $[(1-a)\frac{1}{m_i}+a]A(X)+[(1-a)\frac{1}{m_i}+a\frac{x_{ik}}{x_i}]x_iA'(X)-c=0$. For any $a>0$, this equation implies that in equilibrium every individual in the same group makes the same effort. This explains why we *assumed* equal efforts when $a=0$ even though in that polar case individual effort levels in a group are indeterminate.

[22] In a different game, where players are assigned *exogenously* to groups and do not (as in our game) choose

interval as R1 and its complement as R2. For sharing rules giving more weight than a^* to relative sharing (a in R2), aggregate effort exceeds the social optimum even when all N players form a single group.

If the sharing rule used by all groups were chosen in advance by the N players rather than specified exogenously, what rule would they choose? If each person voted by majority rule on a common sharing rule and the choice was between a rule emphasizing egalitarianism (with a in R1) and a rule emphasizing relative effort (with a in R2), sophisticated voters would unanimously favor the more egalitarian rule. For, anticipating that a mediator would help players coordinate on the payoff-dominant profile in the two-stage game, every voter would foresee that his payoff (1/Nth of maximized social surplus) would be higher under the more egalitarian rule.

6 Conclusion

In this paper, we showed how the free-riding induced in partnerships can be harnessed to increase payoffs when aggregate effort or output would otherwise be excessive. We showed how this idea can be applied to curb excessive extraction from common properties or excessive production from cartels. The same mechanism can achieve the social optimum in innovation tournaments and rent-seeking contests with variable prizes. Inducing limited free-riding may be beneficial in other contexts as well. For example, tips in some restaurants are *separately* solicited by various team members whose combined effort makes a dining experience pleasurable: the maître d', the sommelier, the waiter, the musician, the coat-room provider, the parking valet, etc. In such situations, aggregate effort per customer may be excessive and pooling tips among subsets of these service providers (an increasingly

which group to join, our results imply that there will exist a unique sharing rule common to all groups that would induce socially optimal effort *provided* $n \leq n^*$. That rule, of course, would depend on the exogenous number of groups: $a = \frac{n^*-n}{n-n}$. If $n > n^*$, $a = 0$ generates higher surplus than any larger a but that surplus is still suboptimal.

common practice in restaurants) can be used to raise their net payoffs. Japanese fishermen who have formed partnerships report that pooling revenues reduces congestion and raises price. As we have seen, these are consequences to be expected from such partnerships.

Throughout, we assumed that a partnership had to admit every applicant. It might have been more realistic to assume that members of an existing partnership could deny admission to anyone if opposition to him within the partnership was "sufficiently widespread." This change in assumption would in fact have *increased* the scope of the Partnership Solution. For, every solution we identified as stable would continue to be stable since no one in such solutions has any incentive to join an existing partnership even when assured of admission. But partitions we identified as unstable under our old assumption would become stable under this new assumption. To illustrate, suppose going solo was infeasible and we set up n^* non-solo partnerships some of which differed by two or more members. Such an arrangement could not achieve the first-best under our old assumption because every member of the largest partnership would deviate unilaterally to a smaller partnership with less free-riding. But this same arrangement *would* achieve the first-best under the new assumption since admitting him would be blocked unanimously by existing members who anticipated that expanding the number of partners would stimulate free-riding and would lower each of their payoffs.[23] In assuming that no applicant could be rejected by existing members, therefore, we *understated* the usefulness of partnerships in solving the common-property and cartel problems.

As we have shown there is a temptation in the Partnership Solution to flee one's free-riding partners by going solo. However, other forces often act as a counterbalance. Going solo is sometimes infeasible for technological or legal reasons and, when it is feasible, ceases to be attractive when there are sufficient benefits from team production or fixed costs of

[23]Either the anticipation of rejection by one's new colleagues or of having to carry a disproportionate share of the new partnership's workload (see footnote 9) could explain why the pools of Japanese fishermen persist even with sizes differing by more than one member.

setting up a solo practice. In such circumstances, the Partnership Solution can be used to maximize or at least to raise significantly the aggregate payoff.

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